Life of Fred®

Calculus

Expanded Edition

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What Calculus Is About

I remember standing in the college bookstore at the beginning of my freshman year. I pulled a beginning calculus textbook off the shelf and opened it. What a frightening sight it was.

The pages were filled with strange symbolism like $\int_{x=0}^{2} (4 - x^2)\, dx$ and $\frac{\partial \phi}{\partial x} = M$. It might as well have been in Turkish. No one else in my family had ever studied calculus, so there was no one to give me an overview of what lay ahead. All I was told was that anyone who wanted to study any of the topics that I was even remotely interested in would want to have a grounding in calculus. Even business majors going on for a master's degree were required to study it.

But that didn't tell me what it was. I looked at some of the problems in that old textbook:

11. Find dy/dx for y = sin x.
18. Determine the eccentricity of $(y + 5)^2 + 4(x - 5)^2 = 1$.
22. Solve $y' = \tanh x$

From my trig class I recognized "sin x" and knew that it didn't have theological overtones in this context. But I was still at a loss as to what calculus was about or why I needed to learn it. Was this stuff useful? Would I find a need for it in my everyday life?

Yes. The book you now hold in your hands shows that every aspect of calculus can arise in the course of daily living. If you've ever fallen into a vat of cheese soup (Chapter 19) or tried to run a thousand pounds of ammo through a customs station (Chapter 9) you know what I mean.

So what's calculus? In a sentence:

If it moves at a varying speed,
if it has a curvy shape,
if it has a maximum that you'd like to find, or
if it involves adding up an infinite number of terms,

then you're probably looking at calculus.

A Note to Students

Hi! This is going to be fun.

When I studied calculus my teacher told the class that we could reasonably expect to spend thirty minutes per page to master the material in the old calculus book we used. With the book you are holding in your hands, you will need two reading speeds: thirty minutes per page when you're learning calculus and whatever speed feels good when you're enjoying the life adventures of Fred.

This book has five parts to it:

1. The life adventures of Fred
2. *Your Turn to Play*
3. *Further Ado*
4. Answers
5. Index

Start on the first page of the life adventures of Fred and things will explain themselves nicely.

After 24 chapters you will have mastered all of lower-division (freshman and sophomore) calculus.

A Note to Teachers

This book wasn't written with you in mind. Instead it was created for those who will be learning calculus from it. There are a thousand banal, dignified calculus books with their overwhelming sense of restraint and propriety that present the material as it has always been presented:

definition
theorem
proof
corollary
definition
lemma
theorem
proof
definition
.
.
.

with such a lack of excitement that even a rock would be bored.

There is no reason why the study of calculus should have suffering as its dominant motif—as if our students were children in some Dickens novel. Not withstanding any objections of the American Dental Association, a spoonful of sugar really doesn't do any harm. In the years I taught calculus, I often looked out at the faces of the students and thought to myself that these individuals are not just *Homo habilis* (the toolmaker, the worker) but *Homo ludens* (the playful).

This text will also make your life more pleasant. You haven't been forgotten. Here are some secrets between you and me:

1. *Cities* will be your real friend. Six cities are found at the end of each chapter in the main text. Each city is a set of problems and questions which may take your students 20–30 minutes to work through. You can assign one or more of the cities as homework. In this Expanded Edition all of the answers to the problems in *Cities* are supplied. They will have immediate feedback as to whether they've mastered the material.

2. The main meat of calculus is found in the first section of the book which tells of the life adventures of Fred and in the *Your Turn to Play* which contains lots of completely-worked-out problems. Many instructors assign the problems from the *Your Turn to Play* section. These provide the graded exercises that many beginning students require before tackling the more substantial problems in *Cities*. (*Your Turn to Play* begins at page A-401.)

3. You can make this course as high-powered as you wish by including whatever material you like from the *Further Ado* section. (*Further Ado* begins at page B-441.)

4. Every calculus book leaves out some topics. Very few are left out of *Life of Fred*. (Check the index for your favorite topics.) Furthermore, there are many ideas/topics/themes/presentations that can't be found anywhere else. The rigor is also here with much of it in the *Further Ado* section of the book so that *you* may decide how much of it you'll include in the course you're teaching.

5. On these pages you and your students will find all of calculus folded into the story of the early years of Fred. The topics of calculus arise naturally out of his life experiences.

➢ When Fred and his doll Kingie are playing on the kitchen floor (while his mother makes cookies) he invents a game to play with Kingie. That game evolves into the Mean Value Theorem.

➢ Visiting the buffet at a casino in Oz, Fred and his friends encounter hyperbolic trig functions wherever they turn.

➢ Line integrals arise naturally when Fred does some investigative work at the law library in his attempt to get expelled from kindergarten.

You might enjoy this book as much as your students do!

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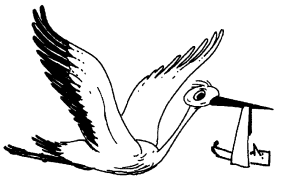
Chapter One

Functions

Once upon a time, a long time ago, on the western slopes of the Siberian Mountains, there lived Fred's parents. However, they weren't called Fred's parents since Fred hadn't been born yet.

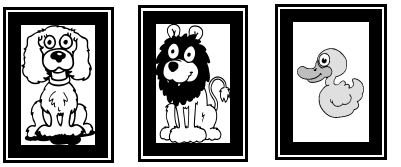
But one day, more recently than a long time ago, the stork delivered Fred. The lucky couple, Mr. & Mrs. Gauss, discovered that they were his parents.



At least Mrs. Gauss (rhymes with "house") thought she was a parent. Staring at Fred, she chattered, "Oh, isn't our baby beautiful!"

Mr. Gauss frowned and said, "He doesn't look a bit like me." Mrs. Gauss didn't get the drift of what her husband was saying. She responded, "Of course he doesn't. He's just a little baby, all red and wrinkly, and, besides, he was very young when he was born." Fred's father rolled back his eyes, turned, and left the room.

Mrs. Gauss carried him around a while and then, not knowing what to do with him, put him back in his crib. She had high hopes that her little tyke would grow up to be a country western singer. After she tucked him in, she handed him a new toy. It was a box with three buttons on it. Each button had an animal printed on it.



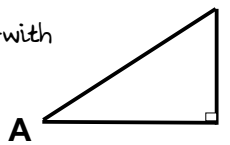
When Fred hit the button with the dog on it, the box sounded, "Bow-wow!" When he tried the lion, "Roar!" The duck, "Quack!" This was Fred's first encounter with the idea of function. He found out that EVERY time he touched the  he heard, "Quack!"

Here is how he summarized in his head what he knew about his animal-toy function:

> 1. There are two sets involved: the set of animal buttons {dog, lion, duck} and the set of animal sounds {Bow-wow, Roar, Quack}.
>
> 2. Every time I hit the lion I get a sound and it's always the same sound.

Fred was fascinated by this idea of function. You start with two sets and for each element of the first set there is exactly one element of the second set which corresponded to it. Fred looked around his study (crib) and invented a new function. His first set contained the things in his crib and the second set was colors. He saw his sheet and that matched up with "white" in the second set. His  matched up with "yellow." The bars on his crib also matched up with "yellow."

Can two different things be "yellow"? Yes. The only critical thing for the idea of a function is that each element in the first set have exactly one **image** in the second set. It's okay if two different elements in the first set have the same image.

Fred thought to himself This is baby stuff! I'm one day old and I should be able to think of a more sophisticated example of a function. He thought of his diaper which was in the shape of a right triangle. He labeled one of the acute angles with the letter A and created the following function: For any acute angle A, draw a diaper—I mean a triangle—with 

one of the acute angles being A.

Then Fred continued measure the length of the side opposite and divide that length by the length of the hypotenuse. When Fred set angle A equal to 35º, the result of using his function (namely, drawing a triangle with a 35º angle and dividing the opposite side by the hypotenuse) gave him a result of 0.5735764. Fred was very good at measuring lengths. He called

this function that he invented the sine function and he wrote $\sin(35^\circ) = 0.5735764$.

But what if he had used a bigger triangle? Would the answer come out differently? No. He knew he'd get the same answer every time since any two right triangles with 35º angles would be similar (something he had read in his geometry book) and similar triangles are triangles in which the sides are proportional.

Now since every element in the first set, which is the set of all acute angles, has a unique image in the second set, Fred knew that he was dealing with a genuine function.

When Mrs. Gauss came in to see how Fred was doing, she found that he had drawn triangles all over his bed sheets.

As she looked down into Fred's study and made little "goo-goo-goo" sounds at him, he said, "Mom, let's play a little game. We'll call it, Guess the Function."

Fred continued, "I'm thinking of a function which I'm going to call 'f' and I'm going to give you some examples and you try and guess what the function is. Are you ready?"

Mrs. Gauss nodded but wasn't sure what Fred was talking about. Then Fred wrote on a sheet:

$f(7) = 15$

$f(3) = 7$

$f(6) = 13$

$f(100) = 201$

Mrs. Gauss looked at what he had written. She looked at it for a long time. Finally she said, "Are you hungry?"

Fred shook his head and gave her some more examples:

f(300) = 601 f(12) = 25

f(0) = 1 f(4.5) = 10

Finally after waiting several more minutes, he gave her the "hint":

f(x) = 2x+1.

He tried to explain to her that a function is any rule which associates to each element of the first set exactly one element of the second set. The object of the game was to guess the rule.

If you would like to play Guess the Function see page A-401 in *Your Turn to Play* in the back of this book. If you are reading this book to learn calculus, and not just enjoy the adventures of Fred, every opportunity for you to take *Your Turn to Play* should be taken. The nice part is that complete solutions are given for all the questions. The *Your Turn to Play* is one of the easiest ways to learn calculus.

His mother left the room humming a tune that didn't seem to have a melody. Fred was alone to think functions some more. He wanted to invent some more of them.

First, I should think of two sets Fred thought and then I'll think of a **mapping** (which is another name for function) between them. He cast his eyes around the room and spotted a huge stack of old country western fan magazines that his mom had stacked up in the corner. We'll let that be the first set. (The first set is called the **domain** of a function.)

From his vantage point in his crib he could see part of the kitchen counter. On it sat a blender, a coffee pot and a mixer. I'll let these be



the elements of my second set. (The second set is called the **codomain** of a function.)

Since a function is ANY rule which associates to each element of the domain, an element of the codomain, Fred could think of hundreds of possible functions:

First example: to every magazine with the letter "R" on the front cover assign those magazines to the coffee pot and all the rest of the magazines assign to the mixer.

Second example: All magazines less than 50 pages long assign to the blender; those with 50–75 pages assign to the coffee pot and the rest to the mixer. Then the **image** of a magazine with 58 pages would be the coffee pot.

Third example: The oldest magazine (which happened to be the July 1952 issue) is assigned to the blender. The youngest magazine is assigned to the coffee pot and all the rest are assigned to the mixer. In this case every element of the codomain was the image of at least one element of the domain. Such a function is said to be **onto** the codomain. (In Fred's first example, that function wasn't onto the codomain since no magazine had the blender as an image.)

Fourth example: Assign all the magazines to the mixer. This, of course, is not onto the codomain. The set of all images of a function is called the **range** of the function. The range of this function is just the mixer. If the range equals the codomain, then the function is onto.

That's a lot of new words and concepts (unless you had them in your algebra classes). Let's turn to *Your Turn to Play* on page A-402 and play with those ideas until they become a bit more comfortable.

(If you want to read more about functions, see page B-441 in *Further Ado* in the back of this book. If you'll read the first three paragraphs on that page, it'll explain what *Further Ado* is all about.)

If you have a yellow highlighter, please color the duck on the previous pages.

It's traveling time. We will have the opportunity to visit six cities in the next several pages. (If you haven't looked at *Your Turn to Play*, these questions will be tougher than they need to be.)

Good news! Answers for the *Cities* are supplied right after the six Cities. If your purpose is to learn calculus, then do the problems first and then check your answers.

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